

**Purdue University**  
**Purdue e-Pubs**

---

Publications of the Ray W. Herrick Laboratories

School of Mechanical Engineering

---

7-2006

# Multi-Patch Near-Field Acoustical Holography and Spatial Resolution Enhancement

Moohyung Lee

J Stuart Bolton

*Purdue University*, [bolton@purdue.edu](mailto:bolton@purdue.edu)

Follow this and additional works at: <http://docs.lib.purdue.edu/herrick>

---

Lee, Moohyung and Bolton, J Stuart, "Multi-Patch Near-Field Acoustical Holography and Spatial Resolution Enhancement" (2006).  
*Publications of the Ray W. Herrick Laboratories*. Paper 96.  
<http://docs.lib.purdue.edu/herrick/96>

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact [epubs@purdue.edu](mailto:epubs@purdue.edu) for additional information.

# **MULTI-PATCH NEAR-FIELD ACOUSTICAL HOLOGRAPHY AND SPATIAL RESOLUTION ENHANCEMENT**

**Moohyung Lee and J. Stuart Bolton**

**July 2006**

## Background

- Constraints of DFT-based NAH
  - A sound field should be sampled with a uniform spacing on a surface of constant coordinate in a separable geometry
  - The hologram surface should extend into a sufficiently large region to avoid spurious effects resulting from the undue truncation of the sound field → Difficult to implement NAH for large scale-structures
- Objective
  - To establish a procedure that is not subject to the latter constraints, thus relieving measurement effort related to the use of DFT
  - An iterative algorithm for recovering missing data and the detailed theoretical background are provided

## Recent Approaches to Patch NAH (1)

- Statistically optimized NAH (SONAH) - *J. Hald*
  - A plane-to-plane propagation is performed in the spatial domain by two dimensional convolution with a propagation kernel
- Helmholtz equation least-squares (HELS) - *S. F. Wu*
  - An assumed solution is expressed by an orthonormal expansion of spheroidal functions that satisfy the Helmholtz equation
  - Solve the Helmholtz equation directly and minimize errors by the least-squares method
- Wavelet-based method - *J.-H. Thomas and J.-C. Pascal*
  - the sound field is decomposed by using multiresolution analysis, and spatial filtering is then performed in a selective way before the wave number spectrum is calculated

## Recent Approaches to Patch NAH (2)

- Method of superposition - *A. Sarkissian*
  - The sound field is approximated by the superposition of fields produced by a number of sources
  - Can be used to either enlarge the finite measurement aperture (extrapolation) or fill the gap in the measurement aperture (interpolation)
- **Iterative patch NAH** - *K. Saijyou and E. G. Williams*
  - The sound field is extended into the region outside the measurement aperture by successive smoothing procedures
  - Available for both DFT and SVD-based NAH
  - Will be described in detail in the present work

# Orthogonal Projection

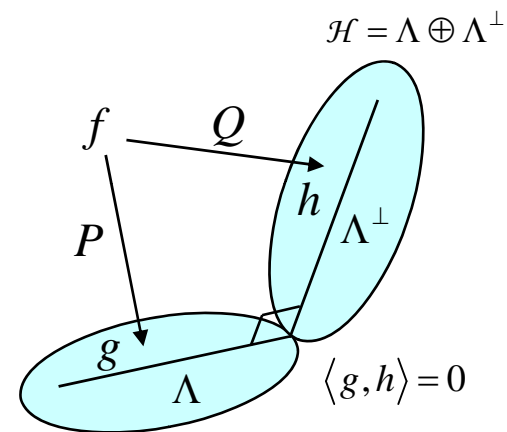
- Consider square integrable functions in a Hilbert space,  $\mathcal{H}$
- Every function in  $\mathcal{H}$  can be decomposed uniquely as

$$f = g + h = Pf + Qf$$

where

$$g \in \Lambda$$

$h \in \Lambda^\perp$ : orthogonal complement



- Properties of the orthogonal projection operator
  - $\|P\|, \|Q\| \leq 1$
  - $P = P^*$  and  $Q = Q^*$  (self-adjoint)
  - $P^2 = P$  and  $Q^2 = Q = 1 - P$

# Data Restoration from Partially Known Information

- When only its projection  $g = P_a f$  onto the known subspace  $\Lambda_a$  is given
- Can  $f$  be reconstructed from  $g$  ?
- To enable the latter task, it is required that an additional constraint be imposed on the nature of the signals
- suppose that  $f$  belongs to the known subspace  $\Lambda_b$  : i.e.,  $f = P_b f$  then,

$$g = P_a f = P_a P_b f = (1 - Q_a) P_b f \dots\dots\dots (1)$$

- $f$  can be uniquely determined from the latter relation if the inverse of  $A = P_a P_b = (1 - Q_a) P_b$  exists
- $\Lambda_b$  and  $\Lambda_a^\perp$  should have only the zero vector in common for solutions to be unique (Youla, D. C., 1978)

# Iterative Method for Missing Data Restoration

- Rearrangement of Eq. (1) gives  $f = Q_a P_b f + g$
- $f$  can be obtained in an iterative way by using a method of successive approximations for finding the fixed point that satisfies the latter relation (method of alternating orthogonal projections): i.e.,

$$f^{(k+1)} = Q_a P_b f^{(k)} + g, \quad k = 1, 2, \dots, \quad f^{(1)} = g \quad \dots\dots\dots (2)$$

- Convergence conditions
  - $\Lambda_b \cap \Lambda_a^\perp = \{\phi\}$
  - $\|Q_a P_b\| < 1$
 } then  $\lim_{k \rightarrow \infty} f^{(k)} = f$
- $f$  in an explicit form

$$f = \sum_{n=0}^{\infty} (Q_a P_b)^n g \quad \Rightarrow \quad f^{(k)} = \sum_{n=0}^{k-1} (Q_a P_b)^n g$$



# Orthogonal Projection Operator (1)

## Sampling Operator

- In NAH applications, the known information is the sound pressure measured over a partial region of the hologram surface
- The measured pressure is expressed in terms of the sound pressure over the complete hologram surface

$$p_m(\vec{r}) = \begin{cases} p(\vec{r}) , & \text{when } \vec{r} \in \Lambda_a \\ 0 , & \text{when } \vec{r} \in \Lambda_a^\perp \end{cases}$$

- The spatial sampling operator

$$p_m = Dp \quad \text{where} \quad D(\vec{r}) = \begin{cases} 1 , & \text{when } \vec{r} \in \Lambda_a \\ 0 , & \text{when } \vec{r} \in \Lambda_a^\perp \end{cases} \dots\dots\dots (3)$$

- No restriction to the spatial distribution of measured data

## Orthogonal Projection Operator (2) Band-limiting Operator

- Related to the additional constraint regarding the nature of signals
- Signals in many practical cases satisfy a certain constraint rather than being arbitrary
- Suppose  $p$  is a function band-limited in  $k$ -space to  $(k_{1,c}, k_{2,c})$
- The band-limiting operator

$$B = F^{-1}LF \quad \text{where} \quad L(\vec{k}) = \begin{cases} 1, & \text{when } \vec{k} \in (k_{1,c}, k_{2,c}) \\ 0, & \text{when } \vec{k} \notin (k_{1,c}, k_{2,c}) \end{cases} \dots\dots\dots (4)$$

- The complete hologram pressure then satisfies  $p = Bp$
- Since high wave number, evanescent sound field components decay quickly, the sound pressure measured on the hologram surface usually (at least weakly) satisfies the latter assumption

## Iterative Formulation

- By combining the two orthogonal projection operators with Eq. (2), the iterative relation for recovering a missing part of a band-limited signal is obtained: i.e.,

$$p^{(k+1)} = (1 - D)Bp^{(k)} + p_m, \quad k = 1, 2, \dots, \quad p^{(1)} = p_m \dots\dots\dots (5)$$

: Papoulis-Gerchberg algorithm

- Since continuous functions cannot be both band-limited and space-limited at the same time, the closed subspace onto which  $p$  is projected by  $(1-D)B$  contains only the zero signal
- Therefore, in continuous-continuous problems, the result obtained by using Eq. (5) converges to the desired solution over a complete region in the absence of noise

## Discrete-Discrete Problems

- Equation (5) is expressed in matrix-vector form
- A unique solution does not exist due to the effects of the artificial truncation of an infinite domain and the discretization of continuous functions
- Since the inclusion of measurement noise is inevitable in practical cases, the band-limitedness assumption is not likely to be strictly valid
- The iteration should be terminated after the finite number of iterations: e.g., when

$$\|p^{(k+1)} - p^{(k)}\| \leq \varepsilon$$

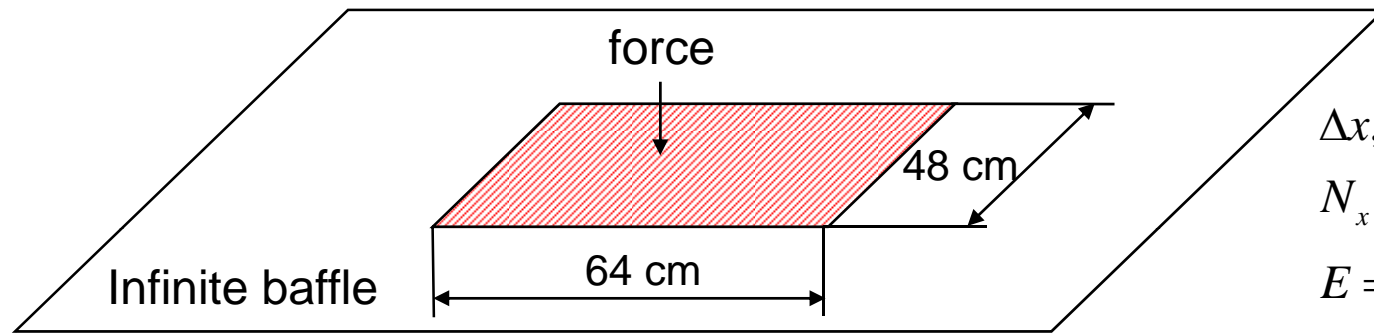
- As a result, the region in which accurate recovery is ensured is limited (usually observed when data is missing in a large contiguous region, i.e., extrapolation problems)

## Comments

- The procedure can be used regardless of the spatial distribution of measured data
  - The convergence rate depends on the latter distribution
    - When data is missing in a large contiguous region (e.g., extrapolation) → slow convergence
    - When data is well distributed (e.g., interpolation) → fast convergence
- The actual bandwidth of a signal is usually unknown
  - In practice, however, the selection of a cutoff of the band-limiting operator within a sensible range usually yields results with reasonable accuracy, and the latter range can usually be identified by examining the wave number spectrum of the patch pressure
  - Is a time-consuming regularization procedure necessary?

# Numerical Simulation Model

Point-driven, simply supported plate within an infinite baffle



$$\Delta x, \Delta y = 0.5 \text{ cm}$$

$$N_x, N_y = 512$$

$$E = 2 \times 10^{11} \text{ Pa}$$

$$\nu = 0.28$$

$$\rho = 7860 \text{ kg/m}^3$$

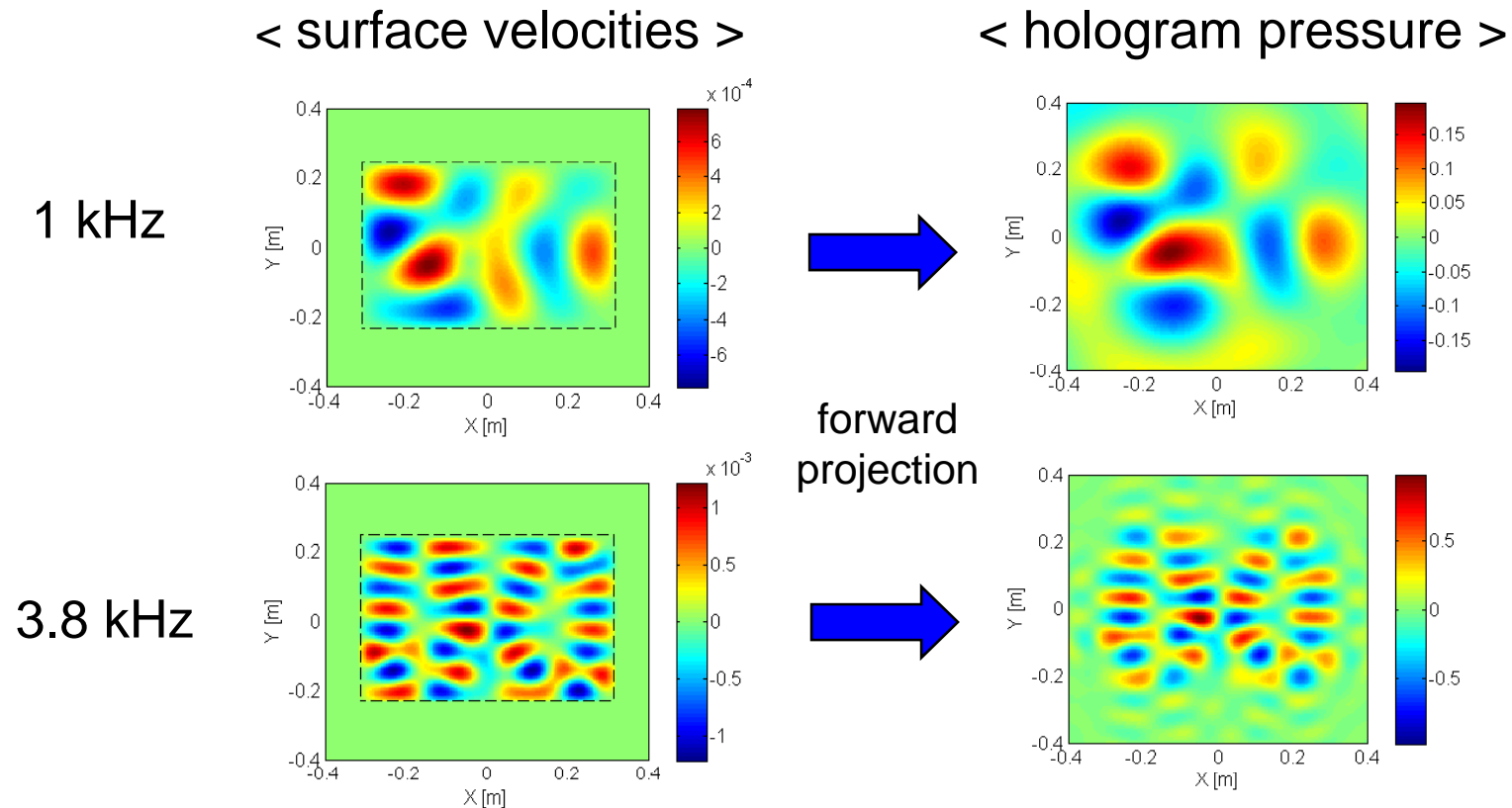
$$h = 5 \text{ mm}$$

$$\dot{w}(x, y, \omega) = \frac{j\omega}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\Phi_{mn}(x_0, y_0) \Phi_{mn}(x, y)}{\omega^2 - \omega_{mn}^2}$$

where

$$\Phi_{mn} = \frac{2}{\sqrt{L_x L_y}} \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right) \quad \omega_{mn} = \sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}} \left[ \left(\frac{m\pi}{L_x}\right)^2 + \left(\frac{n\pi}{L_y}\right)^2 \right]$$

# Surface Velocity and Hologram Pressure

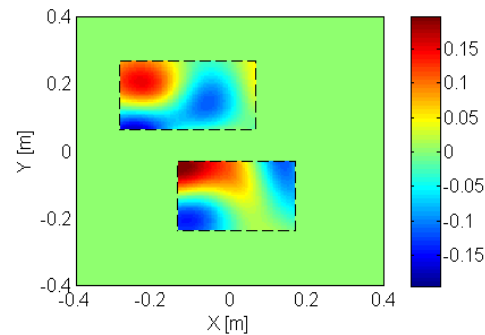


- The height of the hologram surface: 3 cm
- The SNR of the hologram pressures: 40 dB

# Multi-patch Holography (1)

1 kHz

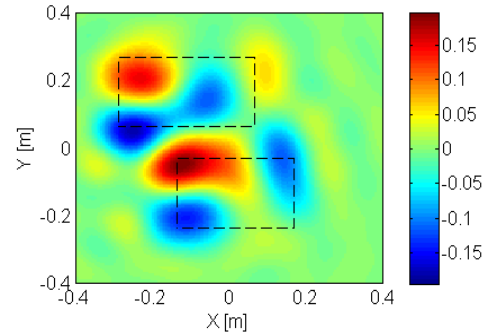
< patch pressures >



400 iter.

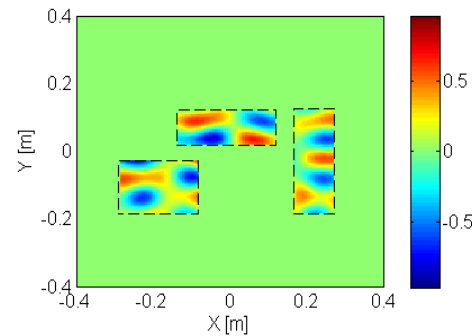


< extended pressures >

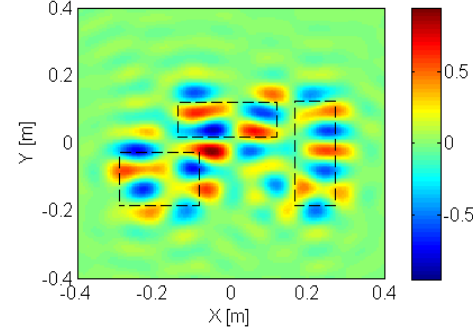


Patch  
extension

3.8 kHz



400 iter.



- The pressures at the locations near the measurement patches were recovered successfully, but the latter could not be achieved over the complete region



## Multi-patch Holography (2)

< A comparison of the surface velocity reconstruction error >

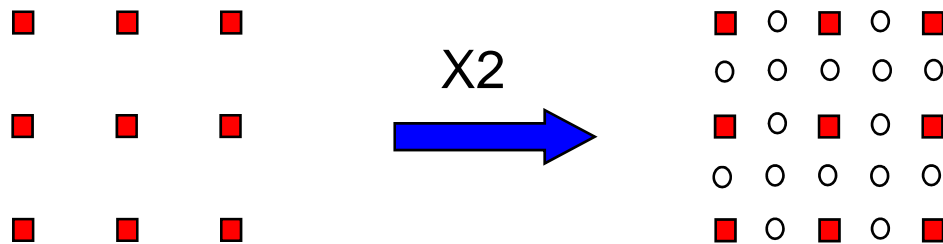
$$error = \|\dot{w}_{rec} - \dot{w}\| / \|\dot{w}\|$$

%	Evaluated over the entire region			Evaluated over the region directly under the patches		
	Complete	Patch	Extended	Complete	Patch	Extended
1 kHz	13.0	119.5	49.2	13.8	104.9	24.8
3.8 kHz	8.9	92.4	51.9	4.7	83.4	10.7

- The use of the extended pressure resulted in significant improvement especially in the region directly under the measurement patches

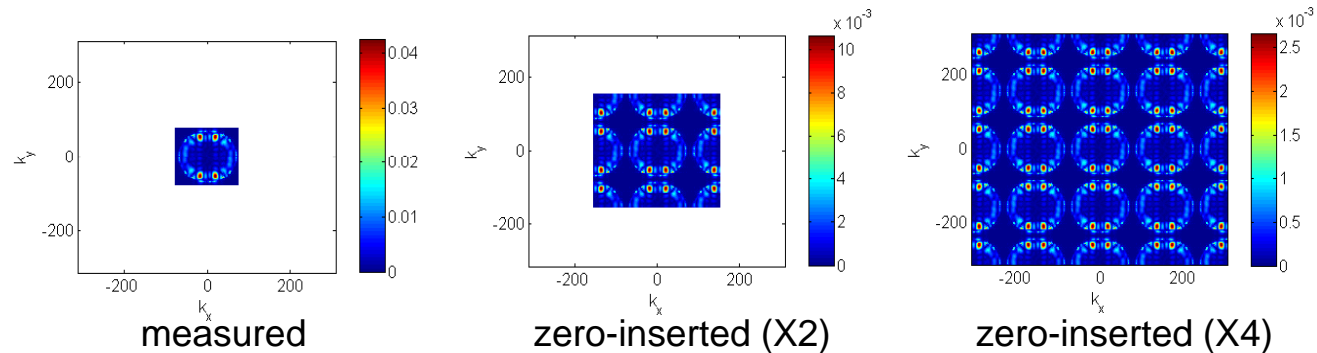
# Spatial Resolution Enhancement (1)

- The initial pressure is prepared by inserting zeros on the uniform grids between the measured points



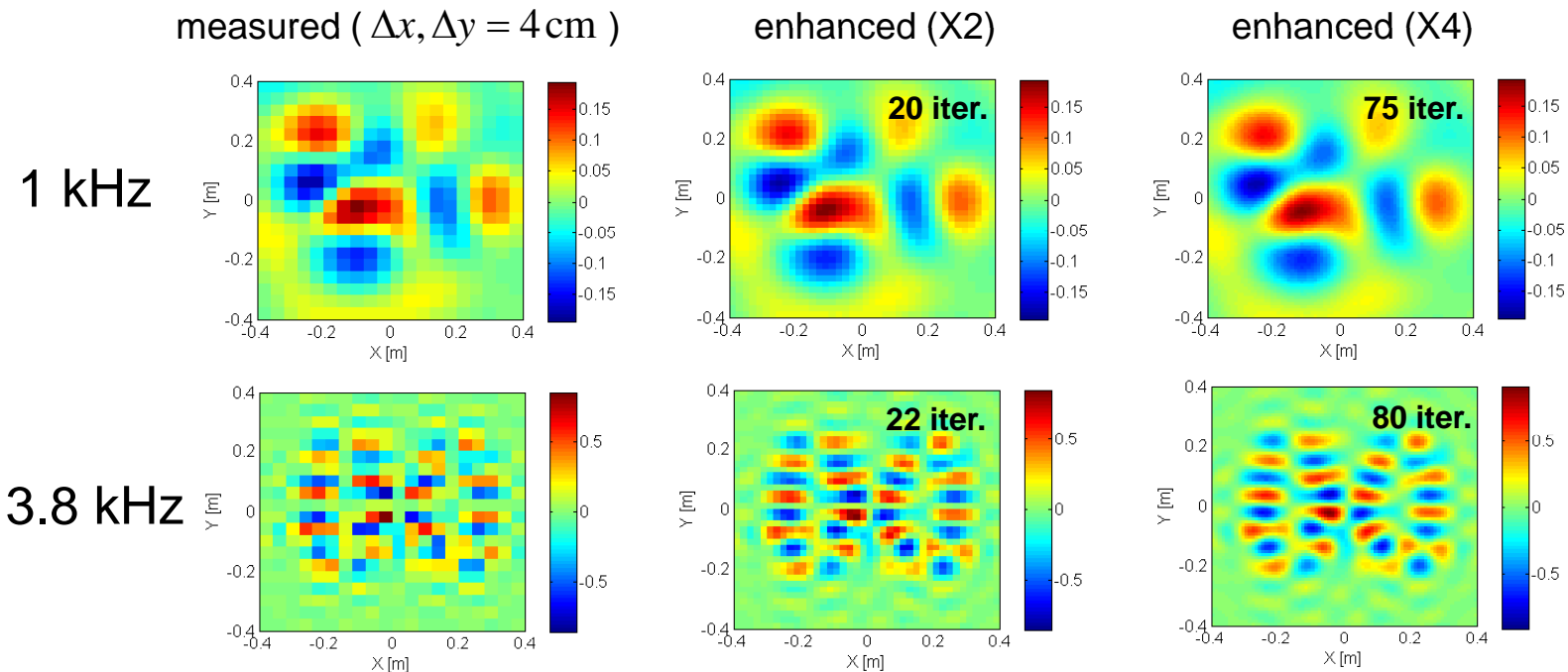
- The actual bandwidth of a signal is “known” in this case

Wave  
number  
spectra  
at 3.8 kHz



# Spatial Resolution Enhancement (2)

< Resolution enhanced hologram pressure >

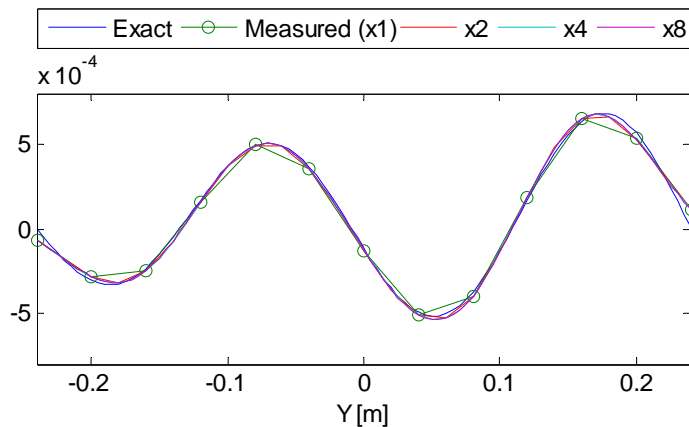


- The results converged far faster than in the extrapolation cases

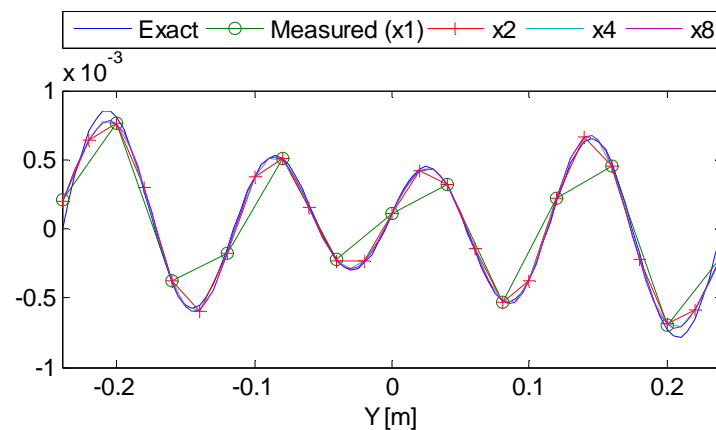
## Spatial Resolution Enhancement (3)

< Comparisons of the surface velocities reconstructed by using the measured and resolution-enhanced hologram pressures >

1 kHz



3.8 kHz



- A larger number of samples per wavelength are necessary to describe the signal shape and locate its peaks correctly

## Conclusions

- An iterative patch NAH algorithm was derived from a method of alternating orthogonal projections
- As assumed when the iterative algorithm was derived, it is important for signals to be band-limited in  $k$ -space for this procedure to be successful
- It was shown that the applications of the procedure described in the present work can be expanded to the hologram pressure measured over arbitrary locations
- The practical limitation of the procedure resulting from the effects of the artificial truncation of an infinite domain and the discretization of continuous functions was discussed
- Numerical simulation results obtained in two cases were presented: i.e., the extrapolation of the hologram pressure measured over multiple, distinct patches, and spatial resolution enhancement by interpolation between measured points